Introduction to the Issue

Shay Allen Logan*

That one philosopher could inspire papers as different from one another as the papers in this issue are is quite remarkable. That one philosopher might inspire papers of this quality is independently quite remarkable. It's a tribute to the genuinely groundbreaking nature of Ross Brady's work that he has managed both.

Unfortunately, this also means it's quite difficult to group the papers in any reasonable thematic way. I've tried to do so nonetheless, and came up with the following four categories. For each category, I note by author which papers I take to belong to it.

- Content semantics and the logic of meaning containment. *Brady, Ferenz/Tedder, Logan, Mendez, Restall*
- Depth Relevance. *Cano-Jorge, Logan, Worley*
- Nonclassical mathematics. *Slaney, Weber/Cano-Jorge*
- Paraconsistent Logics and their Semantic Theories. *Cano-Jorge, Lopez, Mendez, Robles, Zalewski*

There is, of course, a bit of overlap in the above and the categorization is by no means precise. Nonetheless, it should help to situate the work to follow.

In the rest of this introduction, I will aim to give an overview of the contributions the authors in this present issue have made to the topics mentioned above. This will of course be an only partial survey of Brady's total body of work and, even within each of these areas, will only manage to recount some of the broad themes of his work as it applies to the papers in the issue. I'll also note that even though some papers occur in more than one category, each paper will be summarized only once in what follows.

1 Content Semantics and the Logic of Meaning Containment

It's neither new to the study of relevant logic nor particularly insightful to think that something has gone wrong with your story about good inference if it forces you to pretend like it's *perfectly aboveboard* and *in no way at all suspicious* to draw an inference leading from one thing to some other utterly unrelated thing. That's a rather roundabout way of saying that relevance has, essentially *always*, been a thing that folks care about.

*Department of Philosophy, Kansas State University, salogan@ksu.edu

Seeing that it matters, though, is the easy bit. The hard bit is saying what relevance actually *is*. A natural enough place to start is by saying that the sentences A and B are relevant to one another when they share content. Here are two things one might do with this idea:

1. You can reflect on it in a quiet way, and perhaps use it to motivate something like the variable sharing criterion: $A \to B$ ought not be a theorem when A and B don't share variables, or

2. You can shoot it straight into your veins and build a semantic theory that directly assigns contents to sentences and that says the content assigned to $A \to B$ isn't a true content unless the content assigned to A is contained in the content assigned to B.

Brady has opted for the second, straight-into-the-veins, approach. As such things tend to be, the result is invigorating but at times a bit painful. Both of these aspects are at the heart of several of the papers in the issue. Included among these are Brady's own contribution to the issue, which we will discuss first.

1.1 A Normalized Natural Deduction System for the Logic MC

Entailment volume 1 ([1]) famously includes a Fitch-style natural deduction system that is among the earliest pieces of technical machinery in the book, and which recurs in one form or another throughout. Natural deduction systems based on this Fitch system have been prevalent in the literature on relevant logics with [17] being a particularly important example. Brady has made systematic contributions to our understanding of such systems throughout his career—see e.g. [6] for a well-cited example.

As one example of such a contribution, in [8], Brady showed how to normalize a range of these Fitch-style natural deduction systems. As is usually the case, normalization involves removing deviations from a proof where, as he puts it, "deviations are the introduction of a connective and the subsequent elimination of it" ([8, p 48]). The novelty, of course, is that Brady is doing this in Fitch-style rather than Gentzen-style natural deduction systems. In spite of this novelty, the value of normalization remains the same, and Brady uses his normalized natural deduction systems to prove a range of decidability results.

In his contribution to the issue, Brady shows that this idea has a good deal left to give by extending it from the systems examined in [8] to the logic MC. This requires several rather serious modifications of the techniques he had previously applied but the proofs are nonetheless recognizably species of the same genus.

1.2 The Content of Content Semantics

One fact about content semantics that isn't often remarked-on is that it's not just a semantics for propositional relevant logics. It's a full-blown first-order semantics. And it's a full-blown first-order semantics where the machinery for interpreting quantification doesn't look jerry rigged in the way one might expect from the better-known approaches (see e.g. [12, 14, 16, 18, 23]).

For many of us, the straightforwardness of Brady's quantificational apparatus has left us both impressed and suspicious. After all, it's not as if anybody *wanted* the stratification of stratified semantics or the funny algebraic machinery of Mares-Goldblatt semantics. Instead, we simply found that such things were *unavoidable* for the purposes of interpreting quantifiers in a relevant setting.

Of course, in the funny language logicians speak, "I'm impressed and suspicious" really just means "I'd like to understand how that works." In their contribution to the issue, Tedder and Ferenz have finally begun to answer this question by providing two-way interpretations between Mares-Goldblatt semantics and Brady's content models. In so doing, they not only help us come to understand how Brady's models 'work', they also shed light on a few other interesting features of Brady's models including (as they put it) just "how much *syntactic* flavour Brady's content semantics has".

1.3 Reflections on Brady's Logic of Meaning Containment

Restall's paper is a joy to read for a number of reasons, some of which I will cover below. But I'll begin by pointing to a funny coincidence: at least three of the papers in this collection (Logan's, Restall's, and Brady's own) take at least part of their task to be an explication of (some aspect of) what's going on with the meta-rules in the logic of meaning containment MC. Several other papers also touch more-or-less directly on this topic.

Of the three attempts (one of which, I'll note, was my own) to say something insightful about the meta-rules, I found Restall's to be the most informative. At least in part this is because of Restall's framing. The project Restall takes his paper to be engaged in is the project of explaining a few features of MC that (from the perspectives that typically lead folks to consider substructural logics) are unusual. To do this, Restall builds a new class of *algebraic* models of MC—models he aptly names *Brady Algebras*—and proves a soundness and completeness result for MC with respect to Brady Algebras. Much of the rest of the discussion proceeds by examining either particular Brady Algebras or general facts about Brady Algebras that suffice for understanding the features Restall highlighted.

The discussion is illuminating in part because it invites reflection from several different angles. Those interested in understanding *why* MC has the features it has are given, in the algebras and facts about algebras, a way of coming to such an understanding. Those interested in understanding *what* MC is meant to model can reflect on the definition of the algebras and what sorts of structures they are. Those interested in what *sets MC apart* can instead reflect on the particular features Restall highlights and how it is that MC comes to have them.

2 Depth Relevance

A logic L is depth relevant if $A \to B$ is a theorem of L only if there is an atomic formula p and a number $n > 0$ for which there is an occurrence of p in the scope of n conditionals in A and an occurrence p in the scope of n conditionals in B. More briefly, if we say the depth of an atom is the number of conditionals it occurs in the scope of, then L is depth relevant when $A \to B$ is a theorem of L only if A and B share an atom at the same depth.

That a number of the weaker relevant logics are depth relevant—proved by Brady in [5]—is quite surprising. Reflections on depth relevance and its significance have, especially in the last half-decade or so, been impetus for a number of developments both technical and philosophical. The following three papers in the issue explore some of these issues.

2.1 From Depth Relevance to Connexivity

One of the more fun aspects of editing this issue has been seeing the complex network of influences Brady has had on our field. A case in point: Brady's work on depth relevance was the impetus for a range of projects undertaken by Robles and Mendez (see e.g. [20, 22, 19]). These papers, in turns, have influenced a range of other works—including Cano-Jorge's contribution.

At the heart of Cano-Jorge's paper is the following observation: Robles and Mendez have been able to adapt the core idea of Brady's proof that his logic *DR* is depth-relevant to a range of logics that are incommensurable with the standard relevant logics. But another well-known family of logics incommensurable with relevant logics are the connexive logics. So: might we also adapt Brady's method along Robles-Mendez-ian lines so as to produce depth relevant connexive logics? The answer, Cano-Jorge shows, is yes. Showing this is so requires modifications to existing machinery at nearly every step of the way, but the core Brady-Robles-Mendez line remains visible nonetheless.

2.2 Content and Depth Revisited

One of the developments spurred by Brady's work on depth relevance has concerned what's come to be called 'depth hyperformalism'. Both Logan's paper and Worley's paper touch on this issue. Hyperformalism generally is the phenomenon whereby a logic is invariant under some class or other of substitutions broader than just the uniform substitutions. As was shown in [15], many of the logics that Brady showed to be depth relevant in [5] are also *depth hyperformal*—they are invariant under substitutions that are permitted to vary their action with the *depth* of the atom they are acting on.

It turns out—as was proved in [13]—that all depth hyperformal relevant logics are depth relevant. The converse fails—depth hyperformalism is (in the context of relevant logics) the strictly stronger property. Brady's logics of meaning containment, however, possess both properties. This leads to a natural question explored in Logan's contribution: which of the two properties 'matters'? Or, putting the question normatively, in future work on content semantics, what restriction *should we* govern ourselves by: a restriction to depth hyperformal logics or a restriction to depth relevant logics? Logan argues, in part by constructing a depth-varying analogue of Brady's content semantics, that the former restriction is the one that should be in play.

2.3 Proof Invariance

There is, in spite of the discussion in the previous subsection, reason to be worried about depth hyperformalism as a topic of serious study. The point to note is that depth hyperformalism is a meta-level feature of the logics that possess it—stated in full, depth hyperformalism says that whenever a logic happens to have A as a theorem, it will also possess every depth-substitution instance of A as a theorem. It says nothing about e.g. depth-substitution being a valid or meaning-preserving or otherwise 'good' rule. We're naturally left to wonder to what extent depth hyperformalism is worth taking seriously. After all, many other meta-level features of relevant logics seem to be things we clearly *shouldn't* take seriously. For example, traditional relevant logics are all closed under meta-explosion (if A is a theorem and $\neg A$ is a theorem, then everything is a theorem). It does not seem to follow from this that we ought to take meta-explosion to reveal anything particularly deep about these logics.

Worley offers compelling reasons to think that depth hyperformalism, unlike meta-explosion, captures something deep about those relevant logics that possess it. He does so by showing that, for many such logics, not only is their set of theorems closed under depth substitutions, their sets of proofs are as well. To do this, Worley shows that there is a natural (albeit subtle) way of extending the action of depth substitutions on *formulas* to an action on *derivations*. While not definitive, the fact that fact that the action of depth substitutions can be so extended does make it seem much more plausible that depth hyperformalism is a core feature of the relevant logics that possess it.

3 Nonclassical Mathematics

Some large fragment of Brady's work falls into the category of 'logical engineering'. He designs logics, builds logics, proves theorems about logics, and gives philosophical reasons for deciding to do these things in the cases he's concerned with. But logics are, at the end of the day tools. And while it's all well and good to design tools and ensure they're the best tools they can be, Brady hasn't been content to leave things there. He also goes out and uses the tools he's built to do things in the (philosophical) world.

Perhaps the most famous of the things Brady's done with the tools he's built is his construction of nontrivial naïve set theories (see [9, 7, 4]). But this is of a kind with a more general project he's interested in—the project of building a genuinely nonclassical mathematics. Two of the papers in this issue are directly inspired by this part of Brady's work. Both revisit well-known and justly famous bits of mathematics from a nonclassical lens. The project of building a genuinely nonclassical mathematics is an intimidatingly large one. Both authors recognize this, with Weber and Cano-Jorge putting things in what seems to me to be the right way when they say the following:

> Brady's work has taken us far, with farther still to go. We stand on the shoulders of giants—but with universality, we are reaching for the vault of the sky.

3.1 On the Irrationality of the Square Root of 2

Slaney's paper is, he reports, a version of material that appeared in a manuscript and in a typescript, the former of which is labeled "first rough draft". More importantly, they date from 1982 and 1983. This is worth mentioning mostly because of the sound, feel, and texture of Slaney's paper.

There was a time (it was 1973, not too far off from when Slaney wrote most of his paper) when now-infamous sentences like the following could appear in otherwise-respectable journals when one wasn't careful enough to keep the relevantists out:

> And these logics were looked upon with favor by many, for they captureth the intuitions, but by many more they were scorned, in that they hadeth no semantics.

There's really no doubt that the sentence is silly. For some this makes it charming. For others this makes it gauche. Either way, it's a banger. It makes it entirely clear to the reader that something a bit different is about to happen.

Slaney's paper is chock full of bangers of the same sort. Here are a few:

- He describes the principle of contraction as being "at the heart of almost everything deep and devious in logic" and as "critically involved in both proofs and paradoxes of the most fruitful kinds."
- "Anderson and Belnap agree with Church that [the mingle structural rule] violates the spirit of the use criterion, and so they arrange things to ensure that it violates the letter also."
- In describing why the logic RW is odd, he says that it is because "it does not distinguish between the pairs of assumptions A, B and B, A, despite centuries of logical tradition in which major premises have been distinguished from minor ones *as though it mattered*." (emphasis added)

But enough commentary on form. What about content?

Slaney's explicit purpose is to contribute to the sort of project Brady undertook in [7] where interesting fragments of mathematics were founded on weak, non-contractive logics. The specific bit of mathematics Slaney aims to recapture is both a more modest target and a less modest target than Brady aimed at. It is more modest in that Slaney aims 'only' to recapture that the square root of two is irrational (in a weak relevant arithmetic) rather than to rebuild all of set theory. It is less modest in that he has intentionally chosen to focus on a result that very **very** explicitly places contraction at the heart of the proofs one usually sees.

Given this goal, it's a bit surprising to see the rather powerful general conclusions Slaney comes to along the way. As an example, Slaney's Lemma 8 shows that, in his own words, "in a very weak arithmetic, a restricted form of contraction is admissible". What Slaney explicitly shows is that his 'restricted form of contraction' is enough for what he's after. But one has the impression that it would be enough for a great many things one might be after. Slaney is, of course, aware of this and discusses similar themes in his conclusion. One comment worth noting is the final sentence of his paper: "By the time we reach B as a basis for reasoning, negation is banished to the perimiter, and most of mathematics proceeds without it."

3.2 A Note on the Logic of Turing's Halting Paradox

Weber and Cano-Jorge's paper is delightful. It's the only paper of the lot that I'd be comfortable suggesting to a non-logician. Not that it's not technical—it is. It's just that there's *also* both fun personality on show and fun history discussed that would, I think, make it enjoyable for all sorts of folks.

Weber and Cano-Jorge's examination of the halting problem treats it, as they suggest in their title, as a paradox. To me, this was a fairly novel idea on its own. They then extend this novelty by giving the halting problem qua paradox the same treatment we give to most other paradoxes. In particular, they examine different ways to *defuse* the halting problem by either modifying structural rules or modifying the rules governing negation. These solutions lead to different places, but in ways that will feel familiar to those who are used to work defusing other paradoxes in via the same avenues.

4 Paraconsistent Logics and their Semantic Theories

Much of Brady's work has, of course, been concerned with paraconsistent logics. In the course of his career, Brady has defended such logics, proposed novel semantics for such logics, and put such logics to work. We've seen work inspired by his work along these routes above, and some of the papers below lie in these veins as well.

But there is also a critical aspect to some of this work that is addressed in a few of the papers in this section. This critical aspect of Brady's works sees *flaws* in some of the ways relevantists in particular have engaged with the logics they built. Perhaps the best example of this critical work is Brady's paper [10], though threads of this work can be found throughout his corpus. A number of the works below are, directly or indirectly, inspired by this work.

4.1 A 2 Set-up Ternary Relational Semantics for the Companions to Brady's 4-valued logic BN4

Anyone who has spent any serious amount of time thinking about many-valued logics can tell you that there are *lots* of them. Even when attention is restricted to those satisfying a surprisingly large number of conditions, the number of logics one is left with is often large. And it can be hard—often impossible to the uninitiated—to extract insight or intuition from matrices describing truth functions and the like.

So while there's absolutely no doubt that many of these logics *are* interesting and worth studying, it's often quite hard to figure out *why*.

In her paper, Lopez contributes to the continuation of an interesting workaround to this problem first initiated by Brady in [3]. The idea in short is to find a way of presenting some of the many valued logics of interest not using the usual tools of many valued logic, but by instead using the tools of Routley-Meyer semantics. In particular, what Brady initiated (and Lopez has extended) is a trend of using *small* Routley-Meyer models to characterize logics of interest. In the case at hand "small" means, as reflected in the title of her paper, two set-up models.

This is the sort of project that I have a particular soft spot for, so I was glad to see it done. I find it almost always easier to extract philosophically interesting information from a ternary relational model than from a matrix for a truth function. As an example, consider Lopez' description of the Lt1 ternary relation, which relates the following triples:

$$\langle O, O, O\rangle, \langle O, O^*, O^*\rangle, \langle O^*, O, O^*\rangle$$

The first two of these are uninteresting—O, being the base point of the model, is the point under which all other points are closed, so these two have to always happen. That the third can happen *and can be the end of what there is to say* is interesting. In particular, note that O^* is, intuitively, the theory containing exactly the sentences whose negation *isn't* in the logic. That $\langle O^*, O, O^*\rangle$ is in this relation thus tells us roughly that whenever $\neg(A \to B)$ isn't in this logic but A *is*, $\neg B$ isn't.

Let's say that the *counterexample principle* is the statement that $\neg(A \to B)$ is true just if both A and $\neg B$ are. What we learn from $\langle O^*, O, O^*\rangle$ being in the relation governing Lt1 is that it admits an (admittedly loose) rule-analogue of the counterexample principle: $\neg(A \to B)$ is a theorem only if A and $\neg B$ both are. Thus if $\neg(A \to B)$ isn't a theorem but A is, then $\neg B$ isn't.

This is just one example. The point of making it is to say that (at least for people like me) the most exciting part of Lopez' paper is the fact that it gives me philosophically respectable reasons for caring about a range of logics I wanted to care about already but didn't know how to.

4.2 A Quasi-relevant, Connexive 4-valued Implicative Expansion of Belnap-Dunn Logic Defining Material Connexive Logic MC

We touched already on topics concerning connexivity in our discussion of Cano-Jorge's paper. But we did not note there that there was another Bradian connection (connexion?) worth noting: Brady himself has contributed to the study of connexive logics. He has also, in his authoring/editing/shepherding role in [21, 2], given us some of our most in depth examinations of weak relevant logics. In Mendez' paper, these two themes come together in fascinating ways.

There's a tradeoff made early in Mendez' paper that, even though it should be familiar to all of us who work in this area, nonetheless both surprised and pleased me. The tradeoff is this: take the basic relevant logic B, and call B' the logic you get when you replace *its* version of transitivity (namely the prefixing *rule* $A \to B \Rightarrow (B \to C) \to (A \to C)$ and the suffixing rule $B \to C \Rightarrow (A \to B) \to (A \to C)$) with *a conjunctive axiom* of transitivity (namely the axiom $((A \to B) \wedge (B \to C)) \to (A \to C)$). Surprisingly (to me) while B cannot admit good connexive extensions, B' can.

I'll be frank: without proof, I'd not have accepted that this was true. From where I'm sitting, it's a genuinely unexpected result. Luckily, Mendez *has* the proof. In fact, he has more than that, as he gives semantic theories and Hilbert-style axiomatizations for numerous connexive extensions of B'. As is often the case with Mendez' work, his paper opens up vastly more questions than it closes.

4.3 On Three-Valued Logics with the Variable Sharing Property

In the introduction to her paper, Gemma Robles points to an interesting fact: even the parts of Brady's work that he seems to have abandoned (here his work on many-valued logics) remain fruitful. Indeed, as she notes work Brady has mostly abandoned has motivated nearly a decade's worth of work from her and José Méndez.

I view much of what Robles presents (though she herself does not present it this way) as a challenge to contemporary work in relevant logics—a field that is about as tightly associated with Brady as a field can be. The challenge, put simply, is this: justify the bizarre features of your logics and their weird semantic theories. After all, Robles' paper seems to say, there are more than a few logics available with much simpler semantic theories that seem to have many of the desirable features your logics possess. And more: many of the logics relevant logicians tend to concern themselves have *undersirable* features (e.g. undecidability) that the logics Robles presents avoid.

The logics in question are, as the title to her paper suggests, three-valued logics that possess the variable sharing property. In addition, the conditionals in these logics are more-or-less well behaved in ways she carefully lays out. The most important aspect of the well-behaved-ness is a condition she calls 'naturality' that says the conditionals (i) act like the classical conditional when restricted to classical values, (ii) obey modus ponens, and (iii) assign a designated whenever antecedent and consequent are assigned the same value.

There are 11 variable-sharing three-valued logics that possess a conditional that is natural in this sense. Robles identifies a logic she calls b^3 that lies in the intersection of these 11 logics, then characterizes each of the 11 logics in question as an extension of b^3 and also by means of an overdetermined semantic theory. The latter sort of theory, she emphasizes, allows for natural philosophical interpretations of these logics.

I emphasize again that Robles nowhere in her paper presses the line that these logics should be taken as a challenge to work on the traditional family of relevant logics. I nonetheless could not but read the paper as presenting such a challenge. And, given the diversity of the available options and the naturalness of both their axioms and their semantics, it seems to me that the challenge is a serious one.

4.4 Non-Canonical Models of Relevant Logics

One of the more persistent targets of Brady's criticism has been the Routley-Meyer ternary relational semantics. Brady, of course, pairs this criticism with a preferred alternative: his content semantics. But one can take the criticism on its own, in which case other alternatives seem viable as well. Among these, Kit Fine's operational semantics has the longest history, tracing back to [11].

In [16], Fine's semantics was proposed as giving an alternative framework for the entire study of relevant logics. But there are problems with this alternative, several of which are examined in Zalewski's paper.

The short version of the story in [16] is this: relevant logics can be understood as logics of theory building. Understood as such, each Fine-ian operational model ought to be understood as representing a possible space of L-theories for some logic L, and the base point in the model as representing the logic itself.

But recall that a theory for a logic L is just a set of sentences closed under L—that is, a set of sentences t such that if $t \vdash_L A$, then $A \in t$. The *space* of L-theories, then, is just the set of all such sets of sentences, together with some natural pieces of machinery that we'll ignore for now.

The observation at the heart of Zalewski's paper, then, is that any *actual* space of theories will have the following two features:

1. It will have no "double points". That is, there will never be two distinct theories s and t that contain exactly the same sentences. This is ruled out not by anything about theoryhood, but by the mere fact that theories are sets and sets are determined by their members.

2. It will be comprehensive. That is, it will contain *every* set of sentences closed under the logic.

Here's the point: if Fine-ian models are to be understood as models of spaces of theories and *every* actual space of theories has these features, then one would expect Fine-ian models to share these features. But they don't. Zalewski shows, by several rather ingenious constructions, that there are Fine-ian models that have double points and Fine-ian models for which *nearly all* the theories are missing. This poses rather serious challenges for the proposal that we base our philosophical understanding of relevant logics on Fine-ian models understood as models of spaces of theories.

5 Thanks

I'd like to end by issuing a round of "thank you"s. First to the authors because the papers are good. Second to the referees because the refereeing was good. And finally to the (real) editors at the AJL, who deserve much of the thanks for managing to shepherd this project through to the finish line.

Last, to Brady. Your work has inspired my own work and work of the people in this volume in more ways than I've described above. And it has inspired many more people in yet further ways. We one and all look forward to seeing what more both the work you've done and the work you've yet to do will inspire in years to come.

References

[1] Alan Ross Anderson and Nuel D. Belnap. *Entailment: The Logic of Relevance and Necessity, Vol. I.* Princeton University Press, Princeton, N.J., 1975.

[2] Ross Brady, editor. *Relevant logics and their rivals, Volume II, A continuation of the work of Richard Sylvan,Robert Meyer, Val Plumwood and Ross Brady*. Western Philosophy Series, Aldershot, Ashgate, 2003.

[3] Ross T Brady. Completeness proofs for the systems RM3 and BN4. *Logique et Analyse*, 25(97):9–32, 1982.

[4] Ross T. Brady. The simple consistency of a set theory based on the logic CSQ. *Notre Dame Journal of Formal Logic*, 24(4):431–449, 1983.

[5] Ross T. Brady. Depth relevance of some paraconsistent logics. *Studia Logica*, 43(1-2):63–73, 1984.

[6] Ross T. Brady. Natural deduction systems for some quantified relevant logics. *Logique Et Analyse*, 27(8):355–377, 1984.

[7] Ross T. Brady. The non-triviality of dialectical set theory. In Graham Priest, Richard Routley, and Jean Norman, editors, *Paraconsistent Logic: Essays on the Inconsistent*, pages 437–470. Philosophia Verlag, 1989.

[8] Ross T. Brady. Normalized natural deduction systems for some relevant logics I: The logic DW. *Journal of Symbolic Logic*, 71(1):35–66, 2006.

[9] Ross T. Brady. The simple consistency of naive set theory using metavaluations. *Journal of Philosophical Logic*, 43(2-3):261–281, 2014.

[10] Ross T. Brady. Some concerns regarding ternary-relation semantics and truth-theoretic semantics in general. *IfCoLog Journal of Logics and Their Applications*, 4(3):755–781, 2017.

[11] Kit Fine. Models for entailment. *Journal of Philosophical Logic*, 3(4):347–372, 1974.

[12] Kit Fine. Semantics for quantified relevance logic. *Journal of Philosophical Logic*, 17(1):27–59, 1988.

[13] Graham Leach-Krouse, Shay Allen Logan, and Blane Worley. Logic in the deep end. *Analysis*, 84(2):282–291, 2024.

[14] Shay Allen Logan. Notes on stratified semantics. *Journal of Philosophical Logic*, 48(4):749–786, 2019.

[15] Shay Allen Logan. Depth relevance and hyperformalism. *Journal of Philosophical Logic*, 51(4):721–737, 2022.

[16] Shay Allen Logan. *Relevance Logic*. Cambridge University Press, New York, NY, USA, 2024.

[17] Edwin Mares. *Relevant Logic: A Philosophical Interpretation*. Cambridge University Press, New York, 2004.

[18] Edwin D. Mares and Robert Goldblatt. An alternative semantics for quantified relevant logic. *Journal of Symbolic Logic*, 71(1):163–187, 2006.

[19] Gemma Robles and José M. Méndez. Curry's paradox, generalized modus ponens axiom and depth relevance. *Studia Logica*, 102(1):185–217, 2014.

[20] Gemma Robles and José M. Méndez. Generalizing the depth relevance condition: Deep relevant logics not included in R-mingle. *Notre Dame Journal of Formal Logic*, 55(1):107–127, 2014.

[21] Richard Routley, Robert K. Meyer, Val Plumwood, and Ross T. Brady. *Relevant Logics and Their Rivals*. Ridgeview Pub. Co., Atascadero, CA, 1982.

[22] Francisco Salto, Gemma Robles, and José M. Méndez. Curry's paradox, generalized contraction rule and depth relevance. In Konstantinos Boudouris, editor, *Proceedings XXIII world Congress Philosophy*, pages 35–39. Philosophy Documentation Center, 2018.

[23] Andrew Tedder. An algebraic view of the Mares-Goldblatt semantics. *Journal of Philosophical Logic*, 53(2):331–349, 2024.